Count nouns, mass nouns and their transformations: a unified category-theoretic semantics

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1 Introduction

1.1 Prototypical CNs and MNs

All natural languages seem to distinguish at the semantic level between count nouns (CNs) and mass nouns (MNs). Some natural languages, like English, mark the distinction at the syntactic level. Prototypical of CNs is 'dog' and of MNs is 'matter' (in the sense of physical stuff, not in the sense of concern or affair). One syntactic difference is that usually CNs take the plural ('dogs') whereas MNs do not. Other syntactic distinctions relate to the determiners and quantifiers. One can say *a dog*, *another dog*, *many dogs*, *two dogs*, etc.; one cannot correctly say **a matter*, **another matter*, **many matter*, **two matter*, etc. It seems that the distinction in English grammar was introduced by Otto Jespersen [6, p198].

Languages differ in morphology, agreement rules and phrase structure, so one does not expect to find in every natural language a count/mass distinction with the same linguistic correlates as in English. While many European languages are like English in this connection, not all are. Irish and Latin, for example, lack the indefinite article, and so one cannot distinguish CNs from MNs in those languages by the possibility or impossibility of adding the indefinite article to the noun. Japanese has neither definite nor indefinite articles and lacks a uniform way to build plurals. Yet Japanese requires classifiers to licence the application of certain quantifiers such as numerals to nouns. Thus, in order to apply the numeral *ni* ('two') to the noun *inu* ('dog'), the classifier *hiki* is required to form the expression *inu-ga ni-hiki iru* ('There are two dogs'). This classifier is required also for words denoting fishes and insects, although not for words denoting birds, for which the classifier *wa* is required. On the other hand, with almost any count noun denoting inanimate entities the classifier *tsu* is required as in the expression *ringo-o yo-tsu tabeta* ('I ate four apples'). This classifier cannot be used with mass nouns. With mass nouns such as *mizu* ('water'), a different classifier *hai* or *bai* is required to form the expression *mizu-o san-bai (hai) nonda* ('I drank three glasses of water'). This classifier cannot be used with count nouns or with other mass nouns such as *nendo* ('clay'). Thus, the correct use of some classifiers seems to require a mass/count (as well as an animate/inanimate) distinction at the level of nouns. (We owe the details about Japanese to Yuriko Oshima-Takane.)

Incidentally, if we inquire what guides linguists to the decision that there are common nouns in languages other than English, the answer cannot simply be grammar. Grammar varies greatly from language to language as we have just seen. Something other than grammar must be contributing to the decision. We submit that the type of reference for prototypical words plays a major role. We think that across variations in grammar there is a semantic uniformity in the interpretation of at least such prototypical count nouns as *dog* as well as in the interpretation of prototypical mass nouns as *matter* and that this semantic uniformity is a good guide to the relevant grammatical facts in each language. We propose to exploit the semantic uniformity as far as possible.

This should not be taken as an assumption that the perceptual experience of a noun's extension determines whether a noun applied to it is count or mass. It clearly does not. The very same perceptual experience that licenses the application of the MN *gravel* licenses that of the CN *pebble*. Moreover languages vary in what they choose to present in the first instance as mass and what as count. French speakers, for example, apply the CN *meuble* where English speakers would normally apply the MN *furniture*. Notice that *furniture* does supply natural units, or as we say articles of furniture (chairs, tables, lamps, etc.). The importance of the prototypical examples also shows up in work on child language learning. Young children tend to take a new word taught them for a stuff that is like sand in its consistency as a MN, and a word for unfamiliar creatures reminiscent of animals as a CN (see especially [15, 20]).

Pelletier and Schubert [17] document the difficulty in deciding what precisely falls under the classifications mass or count: nouns and noun phrases, adjectives and adjectival phrases, verbs and verb phrases, or adverbs and adverbial phrases? Even if one could settle on the relevant syntactic category or categories, there is still the issue of what precisely in those categories is mass or count: the expressions themselves, their senses, or their occurrences? All this in addition to the problem of how to decide whether something, whatever that something is, is mass rather than count. But notice that to start work on these questions assumes that we understand the mass/count distinction. On the basis of the prototypical examples discussed above, we take the view that this distinction applies to nouns, whether or not it applies to other grammatical categories. Our work is concerned only with the mass/count distinction for nouns, whose solution is presupposed in formulating some of the previous questions. Despite their obvious interest, those questions we do not propose to tackle.

1.2 Cumulative and distributive reference

The usual way of distinguishing MNs from CNs is by specifying semantical properties that MNs have but CNs lack.

Two or more dogs do not together constitute a larger dog; whereas two or more quantities of matter together constitute a larger quantity of matter. It is customary, following Quine [18, p91], to refer to this property of the extension of MNs as *cumulative reference*. Likewise, a (proper) part of a dog is not a smaller dog, whereas a part of matter is matter in the sense that a part of a portion of matter is a smaller portion of matter. Many writers refer to this semantic property of MNs as *homogeneous (or distributed or divided or divisive) reference*. All the authors that we have consulted accept the cumulative reference (as we do) but the property of divisiveness of reference divides them!

What is our position in this debate? Our position can be best formulated by paraphrasing the great Mexican comic Mario Moreno, Cantinflas, 'we are neither for nor against, but quite the opposite'. In fact, if in accordance with the test of cumulative reference we interpret a MN as a sup-lattice M (i.e. a poset such that every subset has a least upper bound), then it makes no sense to ask whether *a part* of an element of M is again an element of M. What could be a part over and above the elements of M? We believe that this discussion presupposes (perhaps inadvertently) a universal substance, a sup-lattice containing every interpretation of a MN as a sub sup-lattice. With such a substance it makes sense to ask whether every element of the universal substance, which is a part of an element of M, is an element of M. We reject the notion of a universal substance, however, and so we are led to reject the whole debate as meaningless. This rejection, however, seems to go too far: take for instance the MN 'footwear'. It applies to a shoe and to any collection of shoes, but does not apply to a proper *part* of a shoe. A shoe's heel is certainly not footwear and a leg of a table is not furniture. Thus, there is a truth of the matter whether a *part* of a shoe is a shoe and a part of a table is a table. All of this seems perfectly meaningful and such knowledge is assumed in the everyday use of the language. Indeed, later we will introduce a notion of substance relative to a system of interpretations of nouns (like 'heel' and 'leg') and these questions will in fact become meaningful. The answer will depend of course on the system of interpretations considered.

Notice that a related, although different question may be formulated meaningfully without invoking a universal substance: is the sup-lattice M atom-less? The reason is that the notion of an atom (or primitive element) is definable in any sup-lattice. We believe that these two questions (whether the part of an element of M is again an element of M and whether M is atomless) have sometimes been confounded, and this has obscured the fact that the first can be formulated only if a universal notion of substance is assumed. At any rate, under the assumption of a universal substance, divisiveness of reference implies that the extensions of MNs are atom-less. This is the only issue that we will address at this point.

Several authors Roeper [19], Lonning [12], Bunt [1], among others, have erected divisiveness of reference into a characteristic of MNs. Bunt [1, pp45-46], for example, allows that in the actual physical extensions of many MNs there are minimal parts, but dismisses the fact as linguistically irrelevant. Quine [18], Parsons [16], Gillon [4] and others reject this claim for the reason that we formulate as follows. It may well be that for many centuries users of the English word *water* have not realized that there are minimal portions of water, namely molecules, whose parts are not water. (Notice how both questions, already discussed, are here confounded). The effective use of the word does not depend on knowledge of that scientific fact. This seems to have led some writers to the conclusion that the facts of the matter have no bearing on the extensions of common nouns. But extension is extension, independent of our knowledge. There is a truth of the matter that there are minimal parts in extensions of some MNs at least: witness the examples of 'footwear' and 'furniture' already discussed. Thus, we do not see reasons to set limitations *a priori* to extensions of MNs. Furthermore, excluding atomic extensions would rule out the possibility of considering plurals as MNs. Not only is there linguistic evidence to consider plurals as MNs, discussed by Carlson and Link (see below), but we will see that the plural construction may be viewed as a basic link between CNs and MNs. The fruitfulness of this approach will be apparent in the paper.

1.3 Categories of nouns and their interpretations

One of the main novelties of our approach is our taking into account the connections between MNs, the connections between CNs and the ways in which MNs can be transformed into CNs and vice-versa. For instance, there is a connection between the MN *iron* and the MN *metal* described in colloquial language by the sentence *iron is metal*, analogous to that between *a dog* and *an animal* that we studied in [9]. Furthermore, there must be some way of connecting the MN *metal* and the CN *a metal* to validate some arguments involving both expressions. This net of connections may be organized in an objective way by means of a system of categories and functors: the nominal theory. This system includes the category $\mathcal{CN}$ whose objects are CNs and whose morphisms are axioms of the form 'a dog is a mammal', 'a mammal is an animal' which may be thought of as a system of identifications. By composing those axioms we obtain the new identification 'a dog is an animal'. In a similar vein, the nominal theory includes the category $\mathcal{MN}$ of MNs such as 'iron', 'metal' and whose morphisms are axioms such as 'iron is metal'. The transformations mentioned before between CNs and MNs are described by functors. One example is the plural formation that takes the CN 'dog' into 'dogs'. Since the extension of this term obviously has the property of cumulative reference, we categorize 'dogs' as a MN. Carlson [2] and then Link [11] called attention to the affinity between plurals and MNs. In fact this affinity even extends to the syntax of both expressions. As an example, plural CNs just like MNs do not take the plural. Also on a par with MNs, such combinations as **a dogs* and **another dogs* are ungrammatical.

We interpret the nominal theory as follows: CNs are interpreted as sets (or kinds). Morphisms between CNs are interpreted as set-theoretical maps. Such maps we call *underlying*. MNs are interpreted as sup-lattices (to be defined precisely later) and morphisms as sup-preserving maps. Functors of the nominal theory are interpreted as functors of the corresponding interpretations. For instance, formation of the plural is interpreted as the power set functor between the interpretations of $\mathcal{CN}$ and those of $\mathcal{MN}$, i.e., between the category of sets, *Sets*, and the category of sup-lattices, *Sl*.

1.4 Grammatical transformations and syllogisms

The nominal theory and its interpretation will be used in this paper for a single goal: to discuss the validity of a sample of eight syllogisms discussed by [17] and involving CNs, MNs and predicables . To achieve this purpose, we need to build semantical counterparts to grammatical transformations of CNs into MNs, MNs into predicables and so on, extending our previous work [9]. As an example, consider the syllogisms

Claret is a wine, wine is a liquid, so claret is liquid

In the first premise, 'claret' is a NP (noun phrase) and 'is a wine' a VP (verb phrase) while in the second, 'wine' is a NP and 'is a liquid' a VP. In the conclusion 'claret' is a NP (as in the first) and 'is liquid' is a VP. The NPs in this example are, however, like PNs (proper names) or descriptions. Cross linguistic evidence suggests that 'claret' is not a PN. In fact, in French, one uses the expression 'le bordeaux' and French does not allow the definite article in front of a PN. This suggests to categorize 'claret' as a descriptive noun phrase (DNP), i.e., as a noun phrase whose interpretation is a member of a kind. As in [9], we consider 'is a wine' as a predicable derived from the term 'wine', a CN as indicated by the occurrence of 'a'. The second premise can be analyzed in a similar way. On the other hand, we will assume that 'is liquid' in the conclusion is a predicable derived from the term 'liquid', an MN as indicated by the absence of 'a'.

At the level of interpretation, these grammatical transformations (or derivations) will take us into computations of colimits of interpretations in our categories to define notions of relative entity, relative substance, etc.

1.5 Category theory versus set theory

A final word about the use of category theory (rather than set theory) in our paper.(For a more thorough discussion, see [13]). A first observation concerns generality: sets themselves constitute a particular category. The use of category theory allows us to formulate semantics that are free of the particular determinations imposed by a too rigid adherence to set theory. From this point of view, we would like to emphasize that the semantics introduced in our paper is one among several possible ones.

A second observation concerns abstractness. Contrary to widespread belief, set-theoretic semantics are more abstract than category-theoretic ones. This is easily understood when we compare usual set-theoretic semantics of CNs and MNs with our category-theoretic semantics. As we pointed out, one of the main novelties of our approach is to take as the basic ingredients of our semantics the connections between dogs and animals, iron and metal, wines and wine, etc. These connections guided the choice of the categories used to interpret CNs and MNs.

To give an example, any sup-lattice is also an inf-lattice and thus, from a set-theoretical point of view, it makes no difference whether one works with sup-lattices or inf-lattices. On the other hand, morphisms of sup-lattices are quite different from morphisms of inf-lattices and the following example shows that underlying relations do not preserve $\wedge$ in general: take a person who has traveled twice, say, and consider the underlying map at the level of sets of persons (which are sup-lattices as well as inf-lattices)

$$2^{PASSENGER} \xrightarrow{u} 2^{PERSON}$$

which associates with a set of passengers the underlying set of persons. Clearly u preserves $\bigvee$ (arbitrary unions in this case). On the other hand, u does not preserve binary $\wedge$: let p_1, p_2 be the two passengers whose underlying person is John. Then $\{p_1\} \wedge \{p_2\} = \emptyset$ and hence $u(\{p_1\} \wedge \{p_2\}) = \emptyset$. But $u\{p_1\} \wedge u\{p_2\} = \{John\}$. Other examples of this kind can be given to justify our choice of categories.

From this point of view, the trouble with set-theoretical semantics is simply this: they are too abstract, since they abstract away these fundamental relations, which are therefore not properly represented (in these semantics). As a consequence, set-theoretical constructions are not constrained in a natural way: there are just too many and when a choice is required, extraneous principles, usually of a pragmatic nature, are brought in to decide the issue. On the other hand, categorical constructions are highly constrained through the use of universal properties: among all possible constructions one is distinguished as satisfying a universal property. Because of this feature, it has been argued that the theory of categories constitutes a theory of concrete universals, set theory being rather a theory of abstract universals [3]. This explains the ubiquity of universal constructions in our paper. In a more speculative way, we are tempted to believe that universal constructions may capture what is universal in the human mind including what is fundamental in human language.

2 The nominal theory

We now describe the *nominal theory*. This theory describes in an objective way the count nouns (CNs), the mass nouns (MNs) and their transformations by means of a system of categories and functors:

$$\begin{array}{c} \mathcal{CN} \\ {\scriptstyle p}\downarrow\uparrow{\scriptstyle u} \\ \mathcal{MN} \end{array}$$

satisfying the following relation: $p \dashv u$, namely p is a left adjoint to u. We will illustrate this relation below. We think of p as 'plural of' and of u as 'a portion of', 'a number of', 'a set of', etc. since there is no unified way, and sometimes simply no way of expressing 'u' at the surface level. The adjointness relation states the equivalence between an axiom of the form $p(\boxed{\text{a portion of meat}}) \xrightarrow{are} \boxed{\text{food}}$ in $\mathcal{MN}$ ('portions of meat are food') and the axiom $\boxed{\text{a portion of meat}} \xrightarrow{is} u(\boxed{\text{food}})$ in $\mathcal{CN}$ ('a portion of meat is a portion of food'). This is clear enough. Sometimes, however, we lack lexical items to express u, as in the equivalence (also obtained from the adjunction) between $p(\boxed{\text{a dog}}) \xrightarrow{are} \boxed{\text{animals}}$ in $\mathcal{MN}$ ('dogs are animals') and the axiom

$\boxed{\text{a dog}} \xrightarrow{is} u(\boxed{\text{animals}})$ in $\mathcal{CN}$ (which could be expressed rather clumsily as 'a dog is a number of animals', with the understanding that this number could be one). Axioms in this relation are said to be 'transpose' of each other.

Notice that under this adjunction, $pu(\boxed{\text{meat}}) \xrightarrow{are} \boxed{\text{meat}}$ ('portions of meat are meat') is an axiom of $\mathcal{MN}$, since its transpose $u(\boxed{\text{meat}}) \xrightarrow{are} u(\boxed{\text{meat}})$ ('a portion of meat is a portion of meat') is an axiom of $\mathcal{CN}$, indeed an identity axiom. Axioms obtained in this way are called 'co-units' of the adjunction. Similarly, $\boxed{\text{a dog}} \xrightarrow{is} up(\boxed{\text{dogs}})$ ('a dog is a number of dogs') is an axiom of $\mathcal{CN}$, since its transpose $p(\boxed{\text{a dog}}) \xrightarrow{are} p(\boxed{\text{a dog}})$ ('dogs are dogs') is an identity axiom of $\mathcal{MN}$. Axioms obtained in this way are called 'units' of the adjunction.

This way of conceptualizing the plural formation seems compatible with that of Jackendoff [5]. The main difference between the two approaches is that we organize nouns into categories ($\mathcal{CN}$ and $\mathcal{MN}$) and so we can consider plural formation as a functor between them, whereas Jackendoff divides nouns into sets : 'individuals' (a *dog*), 'groups' (a *committee*), 'substances' (*water*) and 'aggregates' (*buses, cattles*). Plural formation for him is a map (between some of these sets) which sends, for instance, 'a dog' into 'dogs' and 'a committee' into 'committees'. This is precisely the way our functor acts on objects of the category $\mathcal{CN}$. A further difference concerns his division of nouns. It may be possible to introduce these further divisions in our approach, a question that we have not investigated. We plan to return to this question in a forthcoming paper.

Each category of the nominal theory may be considered as a system of identifications that replaces the '=' of a single sorted theory. These categories seem to be posetal in the sense that there is at most one arrow between two objects, but we will not require them to be so.

$\mathcal{CN}$ is the category whose objects are 'genuine' CNs such as $\boxed{\text{a dog}}$, $\boxed{\text{a mammal}}$, $\boxed{\text{an animal}}$ and whose morphisms are axioms of identification of the form

$$\boxed{\text{a dog}} \xrightarrow{is} \boxed{\text{a mammal}},$$
$$\boxed{\text{a mammal}} \xrightarrow{is} \boxed{\text{an animal}}.$$

The identity morphisms are particular axioms of the form

$$\boxed{\text{a dog}} \xrightarrow{is} \boxed{\text{a dog}}$$

and composition is given by Modus Ponens. For instance from

$$\boxed{\text{a dog}} \xrightarrow{is} \boxed{\text{a mammal}} \text{ and}$$
$$\boxed{\text{a mammal}} \xrightarrow{is} \boxed{\text{an animal}} \text{ we obtain}$$
$$\boxed{\text{a dog}} \xrightarrow{is} \boxed{\text{an animal}}\ .$$

Similarly, $\mathcal{MN}$ is the category whose objects are mass nouns such as $\boxed{\text{water}}$, $\boxed{\text{iron}}$, $\boxed{\text{veal}}$, $\boxed{\text{meat}}$, $\boxed{\text{food}}$ and whose morphisms are of the form

$$\boxed{\text{veal}} \xrightarrow{is} \boxed{\text{meat}}\ ,$$
$$\boxed{\text{meat}} \xrightarrow{is} \boxed{\text{food}}\ .$$

Identity morphisms and composition of morphisms are as above.

The maps p and u are assumed to be functorial. The functoriality of p amounts to say that from

$$\boxed{\text{an A}} \xrightarrow{is} \boxed{\text{a B}}$$

we may conclude that

$$\boxed{\text{As}} \xrightarrow{are} \boxed{\text{Bs}}$$

As an example, from

$$\boxed{\text{a portion of meat}} \xrightarrow{is} \boxed{\text{a portion of food}}$$

we may conclude that

$$\boxed{\text{portions of meat}} \xrightarrow{are} \boxed{\text{portions of food}}$$

The functoriality of u says that from

$$\boxed{\text{M}} \xrightarrow{is} \boxed{\text{N}}$$

we obtain

$$\boxed{\text{a portion of M}} \xrightarrow{is} \boxed{\text{a portion of N}}\ ,$$

In particular, from

$$\boxed{\text{veal}} \xrightarrow{is} \boxed{\text{meat}}$$

we obtain

$$\boxed{\text{a portion of veal}} \xrightarrow{is} \boxed{\text{a portion of meat}} \; ,$$

which agrees with intuition.

Notice that there is a link between, say, $\boxed{\text{meat}}$ and $\boxed{\text{portions of meat}}$. These links are precisely what the functors of the nominal theory are supposed to describe. For instance, from $p \dashv u$ it follows that

$$pu \; \boxed{\text{meat}} \xrightarrow{are} \boxed{\text{meat}}$$

is an instance of the co-unit of the adjunction and hence a morphism of the nominal theory. Recalling that $pu \; \boxed{\text{meat}} = \boxed{\text{portions of meat}}$, this morphism says that 'portions of meat are meat'.

A very interesting feature of the count/mass categorization of nouns is its non-exclusive character. In fact, some nouns such as 'wine' belong to both categories. Thus we may say 'claret is a wine' and 'more wine was served after the dessert'. In the first sentence, 'wine' appears as a CN, whereas it appears as a MN in the second. At the level of the nominal theory, this means that we have two objects:

$$\boxed{\text{a wine}} \in \mathcal{CN} \text{ and } \boxed{\text{wine}} \in \mathcal{MN}.$$

But surely there must be a connection between them. We represent the connection by a morphism of $\mathcal{CN}$

$$\boxed{\text{a wine}} \xrightarrow{is} u \; \boxed{\text{wine}} \; .$$

Equivalently, since p is left adjoint to u, the connection may be expressed by the morphism of $\mathcal{MN}$

$$\boxed{\text{wines}} \xrightarrow{are} \boxed{\text{wine}} \; .$$

These morphisms express that 'a wine is wine' and 'wines are wine' respectively (these assertions are indeed obviously equivalent). In section 3.1 we will interpret the first, as a morphism of the category of Sets. The second, on the other hand, we will interpret as a morphism of the category of Sl, its transpose. Notice that the morphism $\boxed{\text{a wine}} \xrightarrow{is} u\,\boxed{\text{wine}}$ is quite different from $\boxed{\text{a wine}} \xrightarrow{is} \boxed{\text{a wine}}$, a morphism of $\mathcal{CN}$ which expresses that 'a wine is a wine' and which will be interpreted as an identity map in the category of Sets.

The case of wine is not an isolated one. Another example is 'metal'. Thus, we can say 'iron is a metal', but also 'iron is metal'. Once again, the connection between both occurrences of 'metal' is expressed in the nominal theory by a morphism in $\mathcal{CN}$

$$\boxed{\text{a metal}} \xrightarrow{is} u\,\boxed{\text{metal}}$$

Equivalently, we may express the connection by a morphism in $\mathcal{MN}$

$$\boxed{\text{metals}} \xrightarrow{are} \boxed{\text{metal}}$$

Let us notice that beside the endofunctor $pu : \mathcal{MN} \to \mathcal{MN}$ already mentioned and which may be read as 'portions of' or 'numbers of', $up : \mathcal{CN} \to \mathcal{CN}$ is an endofunctor which may be read as 'a set of' (or 'a number of') and whose interpretation will be the power set operation.

Remark 2.0.1 It is interesting to observe that the functor p is not full in general. This means that there might be morphisms $\mathrm{p}(\boxed{\text{an A}}) \xrightarrow{are} \mathrm{p}(\boxed{\text{a B}})$ in $\mathcal{MN}$ which do not come from any morphism $\boxed{\text{an A}} \xrightarrow{is} \boxed{\text{a B}}$ of $\mathcal{CN}$. In more logical terms: the sentences 'A's are B's' and 'an A is a B' are not equivalent in general (although the second implies the first). This fact seems to contradict a widespread belief. As an example, consider 'living room sets are items of furniture' and 'a living room set is an item of furniture'. The first is true, while the second is false.

3 Interpretations of the nominal theory

3.1 Sets, sup-lattices and their connecting functors

Our goal in this section is to define the notion of an interpretation of the nominal theory. As we explained in section 1.3, CNs are interpreted as sets and MNs as sup-lattices. Notice, however, that sets and sup-lattices constitute categories. Indeed, the category of sets, *Sets*, has sets as objects and set-theoretical functions as morphisms. Composition is the ordinary notion of composition between functions. The category of sup-lattices, Sl, has sup-lattices as objects and sup-preserving functions as morphisms. More precisely, an *object* of Sl is a triple $(M, \leq, \bigvee)$ where $(M, \leq)$ is a poset, and $\bigvee : 2^M \longrightarrow M$ associates with every subset of M its supremum. Thus $\bigvee A \leq m$ iff $\forall m' \in A \;\; m' \leq m$. A *morphism* of Sl

$$f : (M, \leq, \bigvee) \longrightarrow (N, \leq, \bigvee)$$

is a set-theoretical map between $f : M \longrightarrow N$ which preserves suprema in the sense that $f(\bigvee A) = \bigvee \exists_f(A)$, where $\exists_f(A) = \{y \in N : \exists x \in A \; (y = f(x))\}$, i.e., the image of A under the function f. Once again, composition is the usual composition between functions.

Remark 3.1.1 In [8] interpretations of CNs are *kinds*, namely sets with a relation of constituency, associating with each member the set of situations of which that member is a constituent. This allows us to study predicables like 'sick' or 'run' which may hold of a member of a kind at a given situation, but fails to hold at another. To simplify the presentation, we will leave situations out of the picture and interpret CNs as sets and MNs as sup-lattices. Furthermore, these simplified notions will suffice for the limited aim of this paper.

There is a connection between these categories that may be described by means of two functors:

$$\begin{array}{c} Set \\ P \big\downarrow \;\; \big\uparrow U \\ Sl \end{array}$$

such that $P \dashv U$. P is the (covariant) power set functor and U is the forgetful functor. More precisely, P is the functor that associates with a set X the sup-lattice PX of the subsets of X, and with a function $f : X \longrightarrow Y$, the function $\exists_f : PX \longrightarrow PY$ defined above. This is clearly a Sl-morphism.

On the other hand, U is the functor which associates with a sup-lattice $(M, \leq, \bigvee)$ the set M and with the morphism $f : (M, \leq, \bigvee) \longrightarrow (N, \leq, \bigvee)$ the function f itself.

We will usually denote by '$O(A)$', '$O(B)$', '$O(C)$', ... the objects of Sl and write '$|O(A)|$' instead of '$U(O(A))$'.

An *interpretation* of the nominal theory in the system of categories just described is given by two functors (both denoted by I) making the following diagram commutative

$$\begin{array}{ccc} \mathcal{CN} & \xrightarrow{I} & Set \\ {\scriptstyle p}\downarrow\uparrow{\scriptstyle u} & & {\scriptstyle P}\downarrow\uparrow{\scriptstyle U} \\ \mathcal{MN} & \xrightarrow{I} & Sl \end{array}$$

in the obvious sense, for instance, $PI = Ip$, etc.

This means that the objects and the morphisms of $\mathcal{CN}$ are interpreted as Sets and morphisms between these Sets, the objects and the morphisms of $\mathcal{MN}$ as objects and morphisms of Sl. Furthermore, the interpretations should 'behave well' with respect to functors of the nominal category. Thus, for instance, $PI = Ip$ means that if the interpretation of $\boxed{\text{a dog}}$ is the set of dogs, then the interpretation of $\boxed{\text{dogs}}$ should be the set of sets of dogs, etc. (It is worthwhile to notice that axioms in the nominal theory are not interpreted as truth-values but as maps.)

Functoriality of $I : \mathcal{CN} \longrightarrow Sets$ means that the interpretation of 'is' in the axiom

$$\boxed{\text{a dog}} \xrightarrow{is} \boxed{\text{an animal}}$$

is a morphism of Sets; intuitively the one that assigns to a dog its underlying animal. Furthermore, the interpretation of 'is' in the axiom

$$\boxed{\text{a dog}} \xrightarrow{is} \boxed{\text{a dog}}$$

is the identity map, etc.

Remark 3.1.2 Lawvere [10] has suggested identifying categories of space with distributive categories and categories of quantity with linear categories. In the Appendix we will show that the target category of the interpretations of CNs, $Sets$, constitutes a category of space whereas the target category of the interpretations of MNs, Sl, constitutes a category of quantity. We feel that any determination of the interpretations of CNs and MNs should have these properties. Ours is just one such determination, chosen mainly for simplicity.

4 Syllogisms and their validity

4.1 Some types of syllogisms

After defining the notion of interpretation we may study validity of all kinds of reasoning involving CNs, MNs, DNPs and predicables, as is usually done in logic. Lacking a systematic way of generating these kinds of reasoning, we will limit ourselves mainly to a few syllogisms that have been studied in the literature ([17]). The notion of validity for syllogisms is the usual Tarski's notion of truth, defined by recursion on complexity of formulas.

(a) Syllogisms involving CNs, PNs and predicables.

These are the usual Aristotelian syllogisms such as

All Greeks are men, all men are mortal, so all Greeks are mortal

In our paper [9] we noticed two problems connected with them: one due to the change of grammatical role of some of the terms, for instance, 'men' appears as part of a predicable in the first premise and as a CN in referring position, subject of the sentence, in the second; the other due to the change of sorting of predicables, for instance, 'mortal' is sorted by the CN 'men' in the second premise and by the CN 'Greek' in the conclusion.

These problems were dealt with in [9] and we will say nothing more about them here beyond remarking that the main idea was to use the colimit of a system of kinds to construct a coincidence relation which in turn provides a semantical counterpart to the transformation of a CN into a predicable. We will extend these notions here to systems of Sl giving semantical counterparts to the transformations of MNs into predicables, essential for the validity of the syllogisms.

(b) Syllogisms involving MNs, DNPs and predicables.

An example of these is

Claret is beer, beer is alcoholic, so claret is alcoholic

Notice that here we have problems similar to those found in the Aristotelian syllogisms: 'is beer' is a VP (or predicable) in the first premise and a NP, in fact a DNP in the second. A further problem concerns the sort of the predicable 'alcoholic': it seems to be sorted by 'beer' in the second but by 'claret' in the conclusion. However, 'claret' is categorized only as a MN in the nominal theory. If this is so, should we allow MNs as well as CNs to sort predicables?

Another example of such a syllogism is

Claret is a beer, beer is a liquid, so claret is liquid

In this syllogism a new difficulty presents itself. In fact, 'liquid' appears as a predicable derived from the CN 'liquid' in the second premiss (as witnessed by the particle 'a'), but as a predicable derived from the MN 'liquid' in the conclusion.

We must give a semantical account of these transformations if we ever hope to test validity of this type of syllogism.

The way to tackle these problems is parallel to that employed with the Aristotelian syllogisms and depends on a notion of coincidence that allows us to transform a MN into a predicable, for instance, from 'beer' to 'to be beer'. These questions will be dealt with in section 4.4.

Remark 4.1.1 *1.* We have given as examples syllogisms whose premises are not necessarily true to avoid falling into the trap of believing that they should be taken as axioms of the nominal theory, thereby limiting validity of syllogisms to a very restricted class.

2. We have given what we believe to be a 'literal' reading of the syllogisms. This does not seem to be the only possible reading. In fact we could also read the first premise as 'claret is a kind of beer'. It is important to keep this in mind when testing validity of syllogisms.

4.2 Coincidence relations and predicates

In the Appendix we will construct notions of entity (a set E) and substance (a sup-lattice $O(S)$), relative to an interpretation, with a map $\theta : E \to O(S)$ and underlying maps $can_X : X \to E$ from each set X which interprets a CN and underlying sup-lattice maps $can_{O(A)} : O(A) \to O(S)$ from each sup-lattice $O(A)$ which interprets a MN.

In turn, these notions will allow us to define two coincidence relations between members of kinds. Both of these notions are reflexive, symmetric and transitive. According to the first, a and b are E-coincident if they have the same underlying entity. As an example, a dog is E-coincident with its underlying animal. According to the second, a and b are $O(S)$-coincident if they have the same underlying substance. As an example, a portion of iron is $O(S)$-coincident with its underlying portion of metal. Because of the existence of the map $E \to O(S)$, E-coincidence implies $O(S)$-coincidence. Thus a dog and its underlying animal have the same underlying substance.

In terms of these notions, we will define the semantical counterpart of the grammatical transformations between CNs, MNs and predicables in section 4.4.

4.3 Grammatical analysis

Following [17], we will limit ourselves to the following list of 8 syllogisms and compare our results with theirs.

1. Claret is a wine, wine is a liquid, so claret is a liquid
2. Claret is a wine, wine is a liquid, so claret is liquid
3. Claret is a wine, wine is liquid, so claret is liquid
4. Claret is a wine, wine is liquid, so claret is a liquid
5. Claret is wine, wine is a liquid, so claret is a liquid
6. Claret is wine, wine is a liquid, so claret is liquid
7. Claret is wine, wine is liquid, so claret is a liquid

8. Claret is wine, wine is liquid, so claret is liquid

Before testing validity of these syllogisms, we need to assign grammatical roles to their terms. As we said in the remark 4.1.1, we will make our reading of them as literal as possible.

First, a general remark to motivate the analysis. Syllogisms are grounded in the nominal theory, in the sense that terms other than predicables belong to the nominal theory and are thus categorized as either a CN or MN. (Some terms may appear in more than one category).

We will assume that the nominal theory categorizes 'claret' as a MN ($\boxed{\text{claret}}$), but 'wine' and 'liquid' as both CNs ($\boxed{\text{a wine}}$, $\boxed{\text{a liquid}}$) and MNs ($\boxed{\text{wine}}$, $\boxed{\text{liquid}}$).

Let us consider the second syllogism. The first premise, 'claret is a wine', is analyzed grammatically as NP + VP. In turn, NP is analyzed as DNP (descriptive noun phrase) with lexical item $\boxed{\text{claret}}$. On the other hand, VP is analyzed as V + NP with lexical items 'is' and $\boxed{\text{a wine}}$. Following [9], the VP 'is a wine' is deduced from the CN 'wine'. The second, 'wine is a liquid', is analyzed similarly. On the other hand the conclusion 'claret is liquid' will be analyzed as DNP + VP, where 'is liquid' is derived from the MN 'liquid'.

4.4 Interpretation and validity

As we said in section 3.1, a CN is interpreted as a kind (or set in this paper), an MN is interpreted as an Sl, a DNP as a member of a kind and a VP as a predicate.

Coming back to the second syllogism, the lexical item $\boxed{\text{claret}}$ in $\mathcal{MN}$, categorized as a DNP should be interpreted as a member of a kind. But what member of what kind? We propose to interpret it as the largest element $1_{O(C)}$ of the sup-lattice $O(C)$ which interprets (the MN) $\boxed{\text{claret}}$, namely as the largest portion of claret, in the kind $|O(C)|$. The predicable 'is a wine' is interpreted as the predicate of being E-coincident with a particular wine. Thus 'claret is a wine' is true iff $1_{O(C)}$ E-coincides with some $w \in W$, where W is the kind of wines. Similarly, 'wine is a liquid' is true iff $1_{O(W)}$ E-coincides with some $l \in L$, where L is the kind of liquids. On the other hand the conclusion, namely, 'claret is liquid' is true iff $1_{O(C)}$ has the property of being liquid. We consider this property as sorted by the CN 'a portion of claret'. Thus the predicate $IS\ LIQUID$ sorted by $u(\boxed{\text{claret}})$ is a function from $I(u(\boxed{\text{claret}}) = |I(\boxed{\text{claret}})| \longrightarrow \Omega$. This means that $1_{O(C)}$ $O(S)$-coincides with a portion $\lambda \in O(L)$, where $O(L)$ is the sup-lattice of portions of liquid, which is the interpretation of the MN 'liquid'. To derive the conclusion, recall that we have a morphism 'wines are wine' in the nominal theory. This morphism is interpreted as a map $f : W \to O(W)$, which to w associates $f(w)$, a portion of wine in $O(W)$. Similarly, the morphism 'liquids are liquid' is interpreted as a map $g : L \to O(L)$.

Since E-coincidence implies $O(S)$-coincidence, from the premises we may conclude

(1) $1_{O(C)}$ is $O(S)$-coincident with $f(w)$

(2) $1_{O(W)}$ is $O(S)$-coincident with $g(l)$

By definition of $1_{O(W)}$, $f(w) \leq 1_{O(W)}$ and this implies the corresponding inequality between the underlying substances:

(3) $can_{O(W)}(f(w)) \leq can_{O(W)}(1_{O(W)})$

since $can_{O(W)}$ is a Sl map.

From (1), (2) and (3) we conclude easily that

(4) $can_{O(C)}(1_{O(C)}) \leq can_{O(L)}(g(l))$

To go further, we need another postulate. We shall assume that an '$O(S)$-portion of liquid is liquid' in the precise sense that $can_{O(L)} : O(L) \to O(S)$ is *downward surjective*: if $s \leq can_{O(L)}(\lambda)$, then there is $\lambda' \leq \lambda$ such that $s = can_{O(L)}(\lambda')$. If this property is satisfied we will say that $O(L)$ *is homogeneous (or distributed or divisive or divided).*

Putting all of this together we have

Proposition 4.4.1 *If the interpretation $O(L)$ of 'liquid' is homogeneous, then the second syllogism is valid*

On the other hand, the first syllogism is not valid, even if $O(L)$ is homogeneous: 'an $O(S)$-portion of a liquid, although liquid, need not be *a* liquid'. In fact, proceeding in this way, we can prove the following

Theorem 4.4.2 *If the interpretation $O(L)$ of 'liquid' is homogeneous, then the syllogisms 2,3,6 and 8 are valid, while the remaining 1,4,5 and 7 are not valid.*

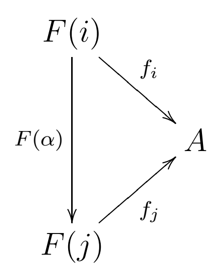
Remark 4.4.3 Our conclusions put us at odd with [17]. Indeed, these authors consider the first syllogism as valid. However we agree on the validity (and non validity) of the remaining ones. The disagreement comes about from the different interpretations assigned to the CN 'a wine'. Contrary to us, they interpret it as 'a kind of wine' and they prove validity from the postulate that 'kinds of kinds are kinds'. A few remarks are in order: their interpretation is higher-order, since they quantify over 'kinds of wine'. Secondly, we do not believe that this rather vague notion allows us to found a logical theory. In particular, the postulate of transitivity is not evident to us.

Appendix

In this section we formulate and prove some mathematical statements used in the main text.

4.5 (Co)cones and (co)limits in a category

Given a functor $F : \mathcal{I} \longrightarrow \mathcal{A}$, a *cocone* for F consists of an object $A \in \mathcal{A}$ and a family $\{F(i) \xrightarrow{f_i} A\}_i$ of morphisms of $\mathcal{A}$ such that for every arrow $i \xrightarrow{\alpha} j \ \in \mathcal{I}$ the diagram



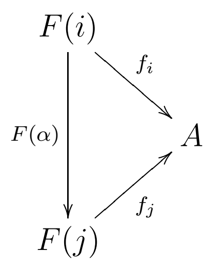
commutes. This cocone will be denoted $(A, \{f_i\}_i)$. Cocones for F constitute a category by defining a morphism

$$\Phi : (A, \{f_i\}_i) \longrightarrow (B, \{g_i\}_i)$$

to be a morphism of $\mathcal{A}$, $\Phi : A \longrightarrow B$, such that $\Phi \circ f_i = g_i$ for all $i \in \mathcal{I}$.

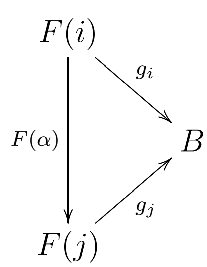
We define *the colimit of* F, *colim*F to be the initial cocone of this category. By spelling this out, *colim*F is a family $\{F(i) \xrightarrow{f_i} A\}_i$ of morphisms of $\mathcal{A}$ such that

1. For every arrow $i \xrightarrow{\alpha} j \ \in \mathcal{I}$ the diagram

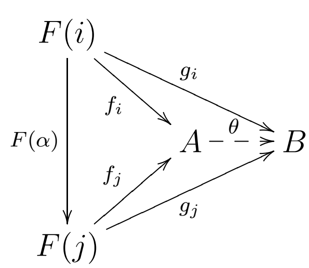
   

   commutes.

2. (Universal Property) Whenever $\{F(i) \xrightarrow{g_i} B\}_i$ is a family of maps such that for every $i \xrightarrow{\alpha} j \ \in \mathcal{I}$ the diagram

   commutes, there is a unique morphism $\theta : A \longrightarrow B \ \in \mathcal{A}$ such that $\theta f_i = g_i$ for all i:

We say that the category $\mathcal{A}$ is *co-complete* if every functor $F : \mathcal{I} \longrightarrow \mathcal{A}$, $\mathcal{I}$ being a small category, has a colimit.

Remark 4.5.1 1. Since terminal objects are unique up to unique isomorphism, we are justified in talking about 'the' colimit of a functor.

2. Dually, we can define *cones* and *limits* in a category by considering $\mathcal{A}^{op}$, rather then $\mathcal{A}$. We say that $\mathcal{A}$ is *complete* if every functor $F : \mathcal{I} \longrightarrow \mathcal{A}$, $\mathcal{I}$ being a small category, has a limit.

4.6 Exactness properties of *Sets* and *Sl*

Theorem 4.6.1 *The category of sets, Sets, is complete, co-complete and distributive in the strong sense that coproducts commute with pull-backs. Furthermore, surjections are stable.*

Proof. All of this is well-known (see, for instance, [14]).

As an example of colimits, let $\mathcal{I} = \mathcal{CN}$ and $F = I : \mathcal{CN} \longrightarrow Sets$ be an interpretation of the category of CNs. The colimit of I may be described as follows:

1. Take the disjoint union $E_0 = \bigsqcup_i I(i)$. Thus an element of E_0 is a couple (i, a) where $a \in I(i)$.

2. Take the smallest equivalence relation $\equiv$ on E_0 generated by the pairs $((i, a), (j, b))$ where there is $\alpha : i \longrightarrow j \ \in \mathcal{CN}$ such that $I(i)(a) = b$.

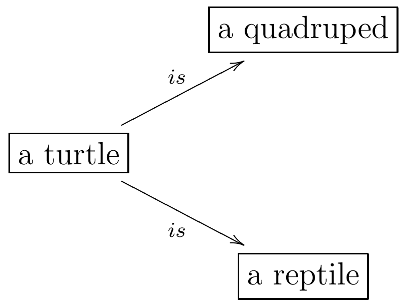
3. Define $E = E_0 / \equiv$ and maps, for each i,

$$I(i) \xrightarrow{u_i} E_0 \xrightarrow{[\ ]} E$$

as follows $a \mapsto (i, a) \mapsto [(i, a)]$ where $[(i, a)]$ is the equivalence class of (i, a) under the relation $\equiv$.

Then $(I(i) \xrightarrow{[\ ]\circ u_i} E)_i$ is the colimit of I .

To explain this construction in one example, assume that



is a diagram in $\mathcal{CN}$ which expresses that ‘a turtle is a quadruped’ and ‘a turtle is a reptile’. In E_0 we have couples like $(\boxed{\text{a quadruped}}\,, q)$, $(\boxed{\text{a turtle}}\,, t)$ and $(\boxed{\text{a reptile}}\,, r)$, where $q \in I(\boxed{\text{a quadruped}}\,)$, $t \in I(\boxed{\text{a turtle}}\,)$ and $r \in I(\boxed{\text{a reptile}}\,)$ respectively. Assume that q is the quadruped underlying the turtle t and r the reptile underlying t . Then $(\boxed{\text{a quadruped}}\,, q)$, $(\boxed{\text{a turtle}}\,, t)$ and $(\boxed{\text{a reptile}}\,, r)$ are among the pairs generating $\equiv$. On the other hand $((\boxed{\text{a quadruped}}\,, q), (\boxed{\text{a reptile}}\,, r))$, although not a generating pair, is in $\equiv$.

Theorem 4.6.2 *The category Sl is complete, cocomplete and linear.*

Proof. This is again well known, (see, for instance, [7]). Recall that a category is linear if it has the following properties

(i) $\mathcal{C}$ has finite products and coproducts (including terminal and initial objects: 1 and 0, respectively)

(ii) the unique morphism $0 \longrightarrow 1$ is an isomorphism with inverse $1 \longrightarrow 0$

(iii) the canonical morphism $A + B \longrightarrow A \times B$ obtained from the morphisms

$$(1_A, 0_{AB}) : A \longrightarrow A \times B$$

and

$$(0_{BA}, 1_B) : B \longrightarrow A \times B$$

is an isomorphism, where 0_{AB} is the composite of $A \longrightarrow 1$, $1 \longrightarrow 0$ and $0 \longrightarrow B$.

We proceed as in [7]. In this category $1 = 0 = (\{*\}, \leq, \bigvee)$

Products are diagrams

$$\begin{array}{ccc} & & (M, \leq_M, \bigvee) \\ & \nearrow^{\pi_X} & \\ (M \times N, \leq, \bigvee) & & \\ & \searrow_{\pi_Y} & \\ & & (N, \leq, \bigvee) \end{array}$$

where $\leq$ is defined pointwise:
$\bigvee_i (x_i, y_i) = (\bigvee_i x_i, \bigvee_i y_i)$, $\pi_x(x, y) = x$, $\pi_y(x, y) = y$.

Coproducts are diagrams

$$\begin{array}{ccc} (M, \leq, \bigvee) & & \\ & \searrow^{i_M} & \\ & & (M \times N, \leq, \bigvee) \\ & \nearrow_{i_N} & \\ (N, \leq, \bigvee) & & \end{array}$$

where $i_M(x) = (x, 0)$ and $i_N(y) = (0, y)$.

Equalizers are diagrams

$$(E, \leq_E, \bigvee) \overset{e}{\hookrightarrow} (M, \leq_M, \bigvee) \underset{g}{\overset{f}{\rightrightarrows}} (N, \leq_N, \bigvee)$$

such that

$$E \overset{e}{\hookrightarrow} M \underset{g}{\overset{f}{\rightrightarrows}} N$$

is an equalizer in Sets, $\leq = (e \times e)^{-1}(\leq_M)$.

Coequalizers are diagrams

$$(M, \leq_M, \bigvee) \underset{g}{\overset{f}{\rightrightarrows}} (N, \leq_N \bigvee) \xrightarrow{q} (Q, \leq, \bigvee)$$

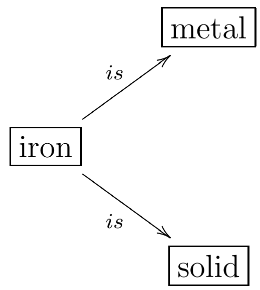
where

$$Q = \{z \in N : \forall x \in M(f(x) \leq z \longleftrightarrow g(x) \leq z)\} \overset{i}{\hookrightarrow} N.$$

and $q \dashv i$. (See [7] for this characterization.)

The universal property of the coequalizer is easy and left to the reader.

As an example of colimit (substance) in Sl assume that $\mathcal{I}$ is the category



where identities have been omitted. Let $I : \mathcal{I} \longrightarrow Sl$ be a functor. We will compute $O(S) = colimI$, following the description of colimits in terms of coequalizers (see [14]), as the coequalizer

$$I(\boxed{\text{iron}}) \underset{g}{\overset{f}{\rightrightarrows}} I(\boxed{\text{metal}}) \times I(\boxed{\text{solid}}) \xrightarrow{q} O(S)$$

where $f(a) = (u(a), 0)$ and $g(a) = (0, v(a))$ and $I(\boxed{\text{iron}}) \xrightarrow{u} I(\boxed{\text{metal}})$, $I(\boxed{\text{iron}}) \xrightarrow{v} I(\boxed{\text{solid}})$ are the interpretations of 'is' in $\mathcal{MN}$.

But $O(S)$ is then obtained by dividing out the coproduct

$$I(\boxed{\text{metal}}) \times I(\boxed{\text{solid}})$$

by the smallest Sl-congruence relation $\equiv$ such that

$$\forall a \in I(\boxed{\text{iron}})\ (u(a), 0) \equiv (0, v(a)).$$

As described in the preceding theorem

$$O(S) = \{(b, c) \in I(\boxed{\text{metal}}) \times I(\boxed{\text{solid}}) : \forall a \in I(\boxed{\text{iron}})(u(a) \leq b \leftrightarrow v(a) \leq c)\}$$

and $q(b, c)$ =the smallest $(b', c') \in O(S)$ such that $b \leq b'$ and $c \leq c'$.

The connection between $Sets$ and Sl is given by the couple of functors P and U. These functors are of a special kind:

Theorem 4.6.3 *The functor P is left adjoint to U. (In symbols: $P \dashv U$).*

Proof. This is well known (see e.g. [14]). The theorem says that there is a natural bijection between $Sl(PX, M)$, the set of sup-preserving maps from PX to M and $Set(X, UM)$, the set of maps between X and the underlying set of M. This is a consequence of the following universal property of the singleton map $X \xrightarrow{\{\,\}} UPX$: for every set map $f : X \longrightarrow U(M)$ there is a unique sup-preserving map $\tilde{f} : PX \longrightarrow M$ such that $U(\tilde{f}) \circ \{\,\} = f$. The proof of this assertion is rather immediate: the definition of the sup-preserving $\tilde{f}$ is forced to be

$$\tilde{f}(A) = \bigvee_{a \in A} f(a)$$

since $\tilde{f}$ must preserve $\bigvee$'s and $A = \bigcup_{a \in A} \{a\}$. (Of course one must prove that the map, so defined, is sup-preserving).

This connection is reflected in a connection between entity and substance:

Theorem 4.6.4 *There is a unique map $\theta : E \longrightarrow O(S)$ such that for each i the following diagram*

$$\begin{array}{ccccc}
I(i) & \xrightarrow{\{\,\}} & |PI(i)| = |Ip(i)| & & \\
 & \searrow^{can_{I(i)}} & & \searrow^{|can_{Ip(i)}|} & \\
 & & E & \xrightarrow[\theta]{} & |O(S)|
\end{array}$$

is commutative

Proof. According to the definition of $E = colimI$, it is enough to show the commutativity of

$$\begin{array}{ccccc}
I(i) & \xrightarrow{\{\,\}} & |PI(i)| & & \\
 & & & \searrow^{|can_{Ip(i)}|} & \\
\downarrow{\scriptstyle I(\alpha)} & & & & |O(S)| \\
 & & & \nearrow_{|can_{Ip(j)}|} & \\
I(j) & \xrightarrow[\{\,\}]{} & |PI(j)| & &
\end{array}$$

for all $i \xrightarrow{\alpha} j \in \mathcal{CN}$. But the diagram may be decomposed into a diagram in $Sets$ and another in Sl:

$$\begin{array}{ccc} I(i) & \xrightarrow{\{\ \}} & |PI(i)| \\ {\scriptstyle I(\alpha)}\downarrow & & \downarrow{\scriptstyle |\exists_{I(\alpha)}|} \\ I(j) & \xrightarrow[\{\ \}]{} & |PI(j)| \end{array}$$

$$\begin{array}{ccc} PI(i) = Ip(i) & & \\ & \searrow^{can_{Ip(i)}} & \\ {\scriptstyle \exists_{I(\alpha)} = Ip(\alpha)}\downarrow & & O(S) \\ & \nearrow_{can_{Ip(j)}} & \\ PI(j) = Ip(j) & & \end{array}$$

The first commutes trivially: $\exists_{I(\alpha)}\{a\} = \{I(\alpha)(a)\}$. The second commutes by definition of $O(S)$. By property (2) of the definition of $E = colim I$, there is a unique $E \xrightarrow{\theta} |O(S)|$ such that $\theta \circ can_{I(i)} = |can_{Ip(i)}| \circ \{\ \}$ which is the conclusion of the theorem.

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